

Quantitative Macro-Labor: Extending the McCall Model with On-the-Job Search

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Announcements

- ▶ Today: On-the-Job Search (Burdett-Mortensen, 1998)
- ▶ You should have logged into the cluster/storage by now.
- ▶ Set up your own folder under “student_folders”. Use the format first initial-last name (i.e., bgriffy).
- ▶ Never put spaces in your directories (use underscores _)
- ▶ Present introduction/outline for research paper next Tuesday.
- ▶ This could be as little as a single slide (see lecture 5 for example).
- ▶ Introduction/outline due following week (2-3 pages).

Running Code on the Cluster I

- ▶ There is code to generate the following datasets:
 1. PSID: you'll need Python and Stata, and run "PSIDMaster.py" after choosing variables.
 2. SIPP: you'll need Stata, run "SIPPMaster.do"
 3. NLSY: pick variables from NLSY website.
- ▶ Run "Bewley1986" Matlab code as a test on cluster
- ▶ Please install the anaconda distribution of Python.
- ▶ Get Matlab from the UAibany software system.

Running Code on the Cluster II

- ▶ Open terminal where your “slurm-file” is located.
- ▶ Type “sbatch slurmFileName” and it will run.
- ▶ Change the email in the slurm file so that I don’t get a bunch of emails.
- ▶ Run “Bewley1986” Matlab code as a test.

Why are Similar Workers Paid Differently?

- ▶ Posed by Dale Mortensen in his book “Wage Dispersion”
- ▶ Abowd, Kramarz, and Margolis (1999): “That... observably equivalent individuals earn markedly different compensation and have markedly different employment histories—is one of the enduring features of empirical analyses of labor markets...”
- ▶ What are some possible reasons?
 1. Ability
 2. Selectivity
- ▶ McCall model: after we control for ability (or whatever), worker selectivity generates all wage dispersion.
- ▶ The more selective workers are, the smaller the dispersion is.
- ▶ Hornstein, Krusell, Violante: Simple models can't match dispersion.

McCall Model Review

- ▶ Model equilibrium characterized by two flow equations and a policy function: unemployment, employment and reservation wage.
- ▶ Unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V - U, 0\} dF(w) \quad (1)$$

- ▶ Employment:

$$rV(w) = w - \delta(V(w) - U) \quad (2)$$

- ▶ Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw \quad (3)$$

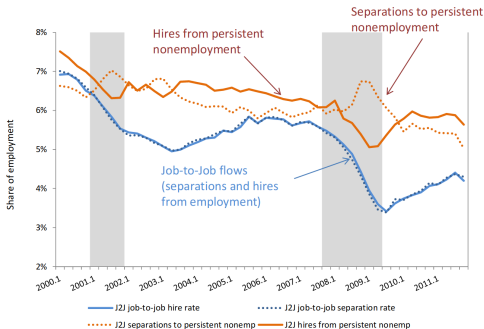
$$w_R = b + \frac{\alpha(1 - F(w_R))}{r + \delta} \int_{w_R}^{\bar{w}} (w - w_R) \frac{dF(w)}{1 - F(w_R)} \quad (4)$$

The Burdett-Mortensen Model

- ▶ What is an (one of many) important and realistic feature of the labor market missing in the standard McCall model?
- ▶ The ability to search while employed.
- ▶ Some statistics:
 1. 50% of all hires are job-to-job hires (Census)
 2. Movement up job ladder accounts for 50% of wage growth for young workers (Topel and Ward, 1992)
 3. 70% of fall in hires during Great Recession was J2J.

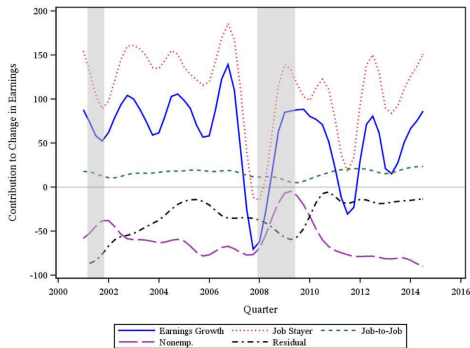
Job-to-Job Flows

- J2J big fraction of job flows (Hahn et al, 2017)



Frictional Earnings Growth

- And earnings growth (Hahn et al, 2017)



The Burdett-Mortensen OTJS Model

- ▶ Extend McCall model to include ability to search while employed.
- ▶ (We'll stick with our notation rather than using theirs)
- ▶ Basic idea:
 1. Workers can be in one of two states: employed or unemployed, with value functions V, U .
 2. Firms post wages, i.e., a given distribution of wages, $w \in [\underline{w}, \bar{w}]$, $w \sim F(\cdot)$.
 3. Unemployed receive job offers at exogenous rate α , no prior info.
 4. Employed receive job offers at exogenous rate λ , no prior info.
 5. Separate two ways: exogenously (rate δ) and via thru OTJS (rate $\lambda[1 - F(w)]$)
 6. Linear utility: $u(c) = b$ or $u(c) = w$.
- ▶ Important assumption: outside offers are unverifiable.
- ▶ i.e., firms cannot respond to outside offers.
- ▶ Intuitively, are there parameter values for which $w_R = \underline{w}$?

Ex Ante Homogeneous Model (“The BM Model”)

- ▶ We'll see two key components to matching flows:
 1. The job-finding rate $\alpha[1 - F(w_R)]$
 2. The worker-finding rate $l(w|w_R, F)$
- ▶ Flow value of unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V(x) - U, 0\} dF(x)$$

- ▶ Employment:

$$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w))$$

- ▶ Over what range will the employed individual accept a job?

Reservation Strategy

- ▶ Reservation wage: $V(w_R) = U$:

$$rV(w_R) = w_R + \lambda \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x) \\ + \delta(U - V(w_R))$$

$$rV(w_R) = rU = w_R + \lambda \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x)$$

$$\Rightarrow rU - rV(w_R) = 0 = b - w_R$$

$$+ (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x)$$

$$\Rightarrow w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x)$$

- ▶ If $\alpha = \lambda$, what is the interpretation?

Reservation Strategy II

- Integration by parts:

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R)) dF(x)$$

$$\begin{aligned} \int_{w_R}^{\bar{w}} (V(x) - V(w_R)) dF(x) &= (V(x) - V(w_R)) F(x) \Big|_{w_R}^{\bar{w}} \\ &\quad - \int_{w_R}^{\bar{w}} V'(x) F(x) dx \\ &= (V(\bar{w}) - V(w_R)) F(\bar{w}) - \int_{w_R}^{\bar{w}} V'(x) F(x) dx \\ &= \int_{w_R}^{\bar{w}} V'(x) [1 - F(x)] dx \end{aligned}$$

$$\rightarrow w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} V'(x) [1 - F(x)] dx$$

Reservation Strategy III

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} V'(x)[1 - F(x)]dx$$

What is the derivative of the employed value function?

$$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w))$$

$$\rightarrow r \frac{\partial V}{\partial w} = dw - \lambda[1 - F(w_R)] \frac{\partial V}{\partial w} - \delta \frac{\partial V}{\partial w}$$

$$\rightarrow \frac{\partial V}{\partial w} = \frac{dw}{r + \delta + \lambda[1 - F(w_R)]}$$



► Thus, the reservation wage is

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} \frac{[1 - F(x)]}{r + \delta + \lambda[1 - F(w_R)]} dx$$

Steady-States

- ▶ Flow unemployment equation:

$$\dot{u} = \delta(1 - u) - \alpha[1 - F(w_R)]u$$
$$u = \frac{\delta}{\delta + \alpha[1 - F(w_R)]}$$

- ▶ Wage distribution?
- ▶ Key feature of this model: can characterize the steady-state wage distribution from model primitives.
- ▶ Define the CDF of employed wages at time t , $G(w, t)$ and population wage distribution $G(w, t)(1 - u(t))$:

$$\frac{\partial G(w, t)(1 - u(t))}{\partial t} = \alpha \max\{F(w) - F(w_R), 0\}u(t) - [\delta + \lambda(1 - F(w))]G(w, t)(1 - u(t))$$

- ▶ $G(w, t)$: prop. of workers receiving no greater than w .

Wage Dispersion in Steady-State

- In steady-state:

$$\frac{\partial G(w, t)(1 - u(t))}{\partial t} = 0 = \alpha \max\{F(w) - F(w_R), 0\} u(t) - [\delta + \lambda(1 - F(w))] G(w, t)(1 - u(t))$$

- For $w \geq w_R$:

$$[\delta + \lambda(1 - F(w))] G(w)(1 - u(t)) = \alpha[F(w) - F(w_R)] u(t)$$

$$\rightarrow G(w) = \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))]} \frac{u(t)}{1 - u(t)}$$

Wage Dispersion in Steady-State

- ▶ Combining unemployment and the wage distribution:

$$\begin{aligned} G(w) &= \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))]} \frac{u(t)}{1 - u(t)} \\ u &= \frac{\delta}{\delta + \alpha[1 - F(w_R)]} \\ \rightarrow G(w) &= \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))]} \frac{\delta}{\alpha[1 - F(w_R)]} \\ &= \frac{\delta(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))][1 - F(w_R)]} \end{aligned}$$

- ▶ Intuitively, what does this say?
- ▶ What does $\lambda = 0$ imply about this distribution? (think about uniform distribution from last class)

The Firm

- ▶ What role does the firm play in all of this?
- ▶ How will it choose which wages to post?
- ▶ Firm profit function:

$$\pi = \max_w (p - w)l(w|w_R, F)$$
$$\pi(w|w_R, F) = (p - w)l(w|w_R, F)$$

- ▶ What is l ? The probability that a firm will meet a worker if it posts a wage w , given
 1. the reservation wage
 2. the distribution of other firm's wages
- ▶ This is an equal profit condition (ie, like the free entry condition).
- ▶ Game against other firms (take F as given).

Worker Finding Rate

- ▶ What is $I(w|w_R, F)$?
- ▶ In steady-state, workers per firm earning a wage w , i.e., probability of finding a worker if you post a wage, w .
- ▶ Inflows:

$$\alpha u + (1 - u)\lambda G(w, t)$$

- ▶ Outflows:

$$(\delta + \lambda[1 - F(w)])$$

- ▶ Define $m = \frac{\text{workers}}{\text{firms}}$. Then,

$$(\delta + \lambda[1 - F(w)])I(w|w_R, F) = m[\alpha u + (1 - u)\lambda G(w, t)]$$

$$\begin{aligned} (\delta + \lambda[1 - F(w)])I(w|w_R, F) &= m\alpha \frac{\delta}{\delta + \alpha[1 - F(w_R)]} \\ &\quad + \frac{m\alpha[1 - F(w_R)]}{\delta + \alpha[1 - F(w_R)]}\lambda G(w, t) \end{aligned}$$

Worker Finding Rate II

- What is $l(w|w_R, F)$?

$$(\delta + \lambda[1 - F(w)])l(w|w_R, F) = m\alpha \frac{\delta}{\delta + \alpha[1 - F(w_R)]} + \frac{m\alpha[1 - F(w_R)]}{\delta + \alpha[1 - F(w_R)]} \lambda G(w)$$

$$l(w|w_R, F) = m\alpha \frac{\delta + [1 - F(w_R)]\lambda G(w)}{(\delta + \lambda[1 - F(w)])(\delta + \alpha[1 - F(w_R)])}$$

$$l(w|w_R, F) = m\alpha \frac{\delta(1 + \lambda \frac{[F(w) - F(w_R)]}{[\delta + \lambda(1 - F(w))]})}{(\delta + \lambda[1 - F(w)])(\delta + \alpha[1 - F(w_R)])}$$

$$l(w|w_R, F) = m\alpha \delta \frac{\frac{\delta + \lambda(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))}}{(\delta + \lambda[1 - F(w)])^2}$$

Equilibrium

- From the paper:

An equilibrium solution to the search and wage-posting game outlined above can be described by a triple (w_R, F, π) such that w_R , the common reservation wage of unemployed workers is the solution to the optimal stopping-time problem out of unemployment, and π is the value function given by the optimal choice of w , and the distribution F is such that

$$(p - w)l(w|w_R, F) = \pi \quad \text{for all } w \text{ on the support of } F$$

$$(p - w)l(w|w_R, F) \leq \pi \quad \text{otherwise}$$

- This is essentially a “free entry” condition.

Analytical Characterizations

- ▶ Assume that $\lambda = \alpha$, i.e., OTJS as efficient as off-the-job.
- ▶ Then, $w_R = b$.

$$l(w|w_R, F) = m\alpha\delta \frac{\frac{\delta + \lambda(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))}}{(\delta + \lambda[1 - F(w)])^2}$$

$$l(w|w_R, F) = \frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

$$\rightarrow (p - w)l(w|w_R, F) = (p - w) \frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

Analytical Characterizations

- In equilibrium, profits are all equal:

$$(p - w)l(w|w_R, F) = (p - w_R)l(w_R|w_R, F)$$

$$(p - w)l(w|w_R, F) = (p - w_R)\frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

$$\rightarrow (p - w)\frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2} = (p - w_R)\frac{m\alpha\delta}{(\delta + \alpha[1 - \cancel{F(w_R)}])^2}$$

$$\rightarrow (\delta + \alpha[1 - F(w)])^2 = \frac{(p - w)}{p - w_R}(\delta + \alpha)^2$$

$$\rightarrow [1 - F(w)] = \frac{(\delta + \alpha)}{\alpha} \left[\frac{(p - w)}{p - w_R} \right]^{\frac{1}{2}} - \frac{\delta}{\alpha}$$

$$\rightarrow F(w) = 1 + \frac{\delta}{\alpha} - \frac{(\delta + \alpha)}{\alpha} \left[\frac{(p - w)}{p - w_R} \right]^{\frac{1}{2}}$$

$$\rightarrow F(w) = \frac{(\delta + \alpha)}{\alpha} \left(1 - \left[\frac{(p - w)}{p - w_R} \right]^{\frac{1}{2}} \right)$$

- We've solved for the offered wage distribution!

Equilibrium Wage Dispersion

- Found that $G(w)$ given by

$$G(w) = \frac{\delta(F(w) - F(w_R))}{[\delta + \alpha(1 - F(w))][1 - F(w_R)]}$$

$$G(w) = \frac{\delta F(w)}{[\delta + \alpha(1 - F(w))]}$$

- And the offer distribution given by

$$F(w) = \frac{(\delta + \alpha)}{\alpha} \left(1 - \left[\frac{p - w}{p - w_R}\right]^{\frac{1}{2}}\right)$$

- Thus, the equilibrium wage distribution is given by

$$G(w) = \frac{\delta \frac{(\delta + \alpha)}{\alpha} \left(1 - \left[\frac{p - w}{p - w_R}\right]^{\frac{1}{2}}\right)}{\left[\delta + \alpha \left(1 - \frac{(\delta + \alpha)}{\alpha} \left(1 - \left[\frac{p - w}{p - w_R}\right]^{\frac{1}{2}}\right)\right)\right]}$$

Next Time

- ▶ Present your introduction/research proposal (could be one slide).
- ▶ Disciplining wage contracts: risk aversion.
- ▶ Between now and then:
 1. Access the campus storage/cluster.
 2. Run some example code.
 3. Create a folder in “user_folders” called first initial-last name (i.e., fvonclownstick).
- ▶ Start your introduction/research proposal write-up. Due following week.