Quantitative Macro-Labor: Responding to Outside Offers with Sequential Auctions

Professor Griffy

UAlbany

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Announcements

- Today: Allow firms to renegotiate wages rather than contract.
- Research proposal/Introduction due 9/24 (Thursday).
- Use feedback from Tuesday presentations.
- Outline of expectations:
  - A (fairly) well-posed research question.
    - Online document has lots of info on this.
    - Don't worry too much about having the perfect question.
  - A discussion of your proposed empirical strategy:
    - I estimate the effect of x on y and (hope to) find z.
    - I use xxx data source.
  - A description of the mechanism you think explains this phenomenon.
    - I show using a model that this is (hopefully) explained by xxxx.
    - The key insight is that in the model, something interacts with something else and causes z.

Contracting Environment in B-M Models

- Standard Burdett-Mortensen:
  - Firms have homogeneous productivity.
  - Cannot respond to outside offers.
  - Contracts stipulate a permanent wage.
  - Distribution of wages posted determined by eqm. wage posting game.
- These contracts are suboptimal,
- Firm would like to retain workers, but artificially restricted:
  1. Cannot respond to outside offers.
  2. Cannot change wage from first offered wage.

Contracting Environment in B-M Models

- Burdett and Coles (2003):
  - Firms have homogeneous productivity.
  - Cannot respond to outside offers.
  - *Contracts specify a value to be delivered over time in expectation*
  - Distribution of determined by eqm. posting game.
- These are upward sloping, backloaded "wage-tenure" contracts.
- These contracts are optimal given the environment:
  1. Firm backloads contracts to reward workers for staying.
  2. Solves the "moral hazard problem" of on-the-job search.
- Really cool paper, hope to cover later in semester.

Empirical Regularities

- We've primarily discussed the theory the last few weeks, but what are the predictions of these models?
- Hoping to revisit this at some point: Burdett and Coles (2003):
  1. Wage profiles are upward sloping.
  2. Wages increase when moving job-to-job.
  3. Job-to-job mobility slows as wages increase.
- What do we observe in the data? (some from Shouyong Shi's notes on directed search)
  1. Wages increase with tenure (Farber, 99) ✓
  2. High wage workers less likely to quit (Farber, 99) ✓
  3. Dispersion among workers with identical tenure
  4. Workers moving *down* the wage ladder.
- Can we use an alternate contracting environment to explain the last two?

Postel-Vinay and Robin (2002)

- Now, a firm *can* respond to outside offers.
- Key ingredients:
  1. Firm heterogeneity in terms of productivity.
  2. Fixed wage contracts.
- The contracts are fixed-wage, but can be *renegotiated*.
- Whenever a worker receives an offer, his current employer tries to convince him to stay.
- Current and offering firm have "auction" over worker (hence sequential auctions).
- Higher productivity firm wins.
- (Note: goal of paper is determining contribution of heterogeneity to wage dispersion, hence two-sided heterogeneity.)

Environment

- Agents:
  - Workers are heterogeneous wrt employment status and ability (fixed).
  - Worker ability: $\epsilon \sim H(.)$.
  - Worker value functions: $V_0(\epsilon), V_1(\epsilon, w, p)$
  - Firms are ex-ante heterogeneous wrt prod., $p \sim F(.), p \in [\underline{p}, \bar{p}]$
- Preferences and Technology:
  - Production of a type-(ϵ, p) match: ϵp
  - Unspecified utility: $u = U(\epsilon b)$, $u = U(w)$.
  - Workers and firms meet at rate λ_0 (unemployed), λ_1 (employed).
  - Exogenous separations, δ, and birth/death μ
- Symmetric discount rate ρ.

Wage Determination

- "Sequential Auctions" a poaching firm bids on a worker against his incumbent firm.
- Wage determination assumptions:
  1. *Firms can vary their wage offers according to worker characteristics.*
  2. *They can counter offers made by competing firms.*
  3. All offers are take-it-or-leave-it.
  4. Contracts are long-term and can be renegotiated by mutual agreement.
- Take-it-or-leave-it offers are the result of game played between firms.
- This can generate *within-firm* variation in wages based on luck.
- Some workers happen to run into other firms more often $\rightarrow$ higher wages.

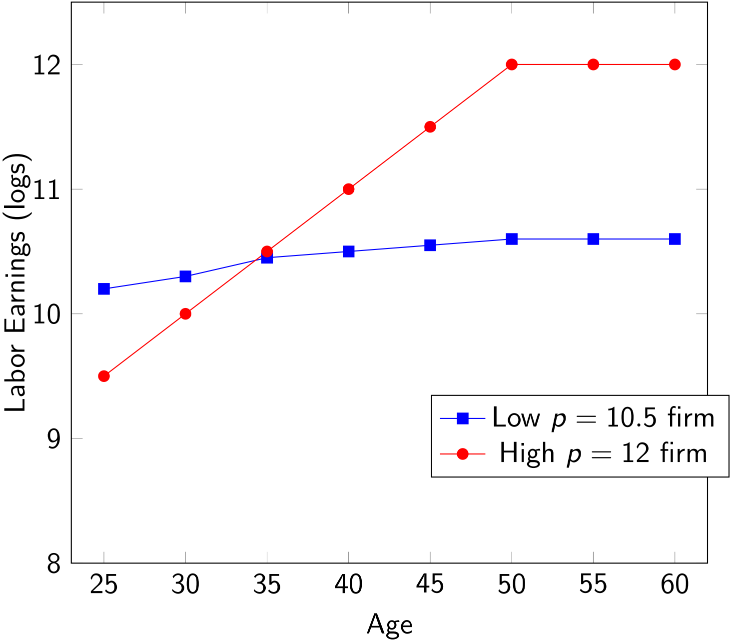
Unemployed Value Function

- Unemployed flow value:

$$(\rho + \mu + \lambda_0)V_0(\epsilon) = U(\epsilon b) + \lambda_0 \int_{p_R}^{\bar{p}} V(\epsilon, \phi_0(\epsilon, x), x) dF(x)$$

- What is $\phi_0(\epsilon, p)$? Function mapping $\phi_0 : R_{\epsilon \times p} \to R_+$ heterogeneity to wages.
- Firms make take-it-or-leave-it offers.
- What is the
  1. Wage offered to firms?
  2. Reservation "mpl" (they mean p)?
- What does take-it-or-leave-it offers mean about a worker's bargaining power?

Employed Reservation Strategy



Equilibrium Wages

- Worker with state (ϵ, w, p)
- What is the maximum the incumbent firm p, could pay? $w = \epsilon p$.
- Worker could run into the following firms characterized by their productivity:
  1. Firm $p' \leq \frac{w}{\epsilon}$:
     - p' so low that highest wage less than current wage. $\epsilon p') \leq w$
  2. Firm $p' < p$, but $\epsilon p' > w$:
     - p' firm cannot outbid p firm, but bids wage up.
  3. Firm $p' > p$:
     - Incumbent firm cannot match poaching firm. Wage falls to compensate poaching firm for future wage increases.

Equilibrium Wages

- ϕ: wage that makes worker indifferent given ϵ and productivities p, p'. Second argument is always $\tilde{p} > \hat{p}$.
- Define a productivity threshold q such that

$$\phi(\epsilon, q(\epsilon, w, p), p) = w$$

- q is the lowest productivity firm $p \in [\underline{p}, \bar{p}]$ from which an offer can impact the wage.
- Corresponding continuation values and probabilities:
  1. Firm $p' \leq \frac{w}{\epsilon}$:
     - Probability: $F(q(\epsilon, w, p))$, CV: $V(\epsilon, w, p)$.
  2. Firm $p' < p$, but $\phi(\epsilon, p', p) > w$:
     - $F(p) - F(q)$, $V_{t+1} = V(\epsilon, \phi(\epsilon, p', p), p) = V(\epsilon, \epsilon p', p')$
  3. Firm $p' > p$:
     - $1 - F(p)$, $V_{t+1} = V(\epsilon, \phi(\epsilon, p, p'), p') = V(\epsilon, \epsilon p, p)$

Wage Cuts while Moving up Ladder

- As an example, consider two firms with income growth rates γ_1 and γ_2, $\gamma_2 > \gamma_1$.
- You are currently employed by firm 1 at a wage y_1, and firm 2 is offering you y_2.
- You must work for whoever you pick permanently, and you are maximizing lifetime income with discount rate β.
- Lifetime income:

$$\sum_{t=0}^{\infty}((1+\gamma_j)\beta)^t y_j$$

- Present values:
  1. Firm 1: $\frac{y_1}{1-\beta(1+\gamma_1)}$
  2. Firm 2: $\frac{y_2}{1-\beta(1+\gamma_2)}$
- In this case, what we are saying is that firm 2 would pick y_2 st

$$y_2 = \frac{y_1(1-\beta(1+\gamma_2))}{1-\beta(1+\gamma_1)}$$

Employed Value Function

- Flow value of employment ($q = q(\epsilon, w, p)$):

$$(\rho + \delta + \mu)V_1(\epsilon, w, p) = U(w) + \delta V_0(\epsilon) + \lambda_1 \int_q^p V(\epsilon, \phi(\epsilon, p, x), p) dF(x)$$

$$(\rho + \delta + \mu)V_1(\epsilon, w, p) = U(w) + \delta V_0(\epsilon) + \lambda_1 \int_q^p [1 - F(x)] \frac{\partial V}{\partial \phi} \frac{\partial \phi}{\partial x} dx$$

- How do we find $\frac{\partial V}{\partial \phi}\frac{\partial \phi}{\partial x}$? From q and p, any competing offer $\rightarrow V(\epsilon, \phi, p) = V(\epsilon, \epsilon x, x)$.

$$\rightarrow V(\epsilon, \epsilon p, p) = \frac{U(\epsilon p) + \delta V_0(\epsilon)}{\rho + \delta + \mu}$$

$$\rightarrow (\rho + \delta + \mu)V_1(\epsilon, w, p) = U(w) + \delta V_0(\epsilon) + \frac{\lambda_1 \epsilon}{\rho + \delta + \mu} \int_q^p [1 - F(x)] U'(x) dx$$

Reservation Strategies

- Employed reservation strategy:

$$\begin{aligned} V(\epsilon, \phi(\epsilon, p, p'), p') &= V(\epsilon, \epsilon p, p) \\ V(\epsilon, \phi(\epsilon, p, p'), p') - V(\epsilon, \epsilon p, p) &= 0 \\ \rightarrow V(\epsilon, \phi, p') - \frac{U(\epsilon p) + \delta V_0(\epsilon)}{\rho + \delta + \mu} &= 0 \end{aligned}$$

- From earlier: $V(\epsilon, \epsilon p, p) = \frac{U(\epsilon p)+\delta V_0(\epsilon)}{\rho+\delta+\mu}$.

$$\begin{aligned} \underbrace{U(\phi(\epsilon, p, p'))}_{\textit{Poaching Utility}} = &\underbrace{U(\epsilon p)}_{\textit{Incumbent Utility}} \\ &- \underbrace{\frac{\lambda_1}{\rho + \delta + \mu}}_{\textit{Offer Arrival}} \underbrace{\int_p^{p'} [1 - F(x)]\epsilon U'(\epsilon x) dx}_{\textit{Wage Growth Utility}} \end{aligned}$$

- Inverting this function yields the reservation strategies.
- Identical argument for unemployed workers.

Decomposition

- Conveniently, reservation equation log-linearizes for different utility functions (CRRA, $U(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$):

$$
\begin{aligned}
ln(\phi(\epsilon, p, p')) &= ln(\epsilon) + ln(\phi(1, p, p')) \\
ln(\phi(\epsilon, p, p')) &= ln(\epsilon) \\
&\quad + \frac{1}{1-\alpha} ln(p^{1-\alpha} - \frac{\lambda_1(1-\alpha)}{\rho+\delta+\mu} \int_p^{p'} [1-F(x)] x^{-\alpha} dx), \alpha \neq 1 \\
ln(\phi(\epsilon, p, p')) &= ln(\epsilon) + ln(p) \\
&\quad - \frac{\lambda_1}{\rho+\delta+\mu} \int_p^{p'} [1-F(x)] \frac{dx}{x}, \alpha = 1
\end{aligned}
$$

- Here, $ln(\epsilon)$ is the *worker* effect.
- And $ln(\phi(1, p, p'))$ is the labor market history effect.

Steady-State Equilibrium

- They are interested in the cross sectional dispersion of wages, so they focus on the steady-state.
- "The steady state assumption implies that inflows must balance outflows for all stocks of workers defined by a status (unemployed or employed), a personal type ϵ, a wage w, and an employer type p."
- The equilibrium objects are
  1. Reservation strategies for each worker over firm productivities, given the distributions and prices.
  2. Wage function for for each tuple (ϵ, p, p') with $p' = b$ for unemployed, given the distributions.
  3. Flow equations that balance according to the statement above.
- They derive the distributions in the paper.

Log-Wage Variance

- We will define a firm by its productivity "type"
- Recall definition of conditional variance:

$$V(x) = E[V(x|y)] + V[E(x|y)]$$

- The log-linearity of wages is very useful!

$$\begin{aligned} ln(\phi(\epsilon, q, p)) &= ln(\epsilon) + ln(\phi(1, q, p)) \\ \rightarrow E[ln(\phi(\epsilon, q, p))|p] &= E[ln(\epsilon)] + E[ln(\phi(1, q, p))|p] \\ \rightarrow V[ln(\phi(\epsilon, q, p))|p] &= V[ln(\epsilon)] + V[ln(\phi(1, q, p))|p] \end{aligned}$$

- Then the total variance of wages is given by

$$\begin{aligned} V(ln(w)) &= V(ln(\epsilon)) + V(E[ln(w|p)]) + (E[V(ln(w|p))] - V(ln(\epsilon))) \\ &= \underbrace{V(ln(\epsilon))}_{Individual} + \underbrace{V(E[ln(\phi(1, q, p))|p])}_{Between\ Firm} \\ &+ \underbrace{E[V(ln(\phi(1, q, p))|p)]}_{Within\ Firm\ non-individual} \end{aligned}$$

Empirical Analysis

- They use a matched employer-employee dataset from France.
- They estimate the model, and then use simulated data to decompose the size of the worker effect, the firm effect, and the labor market effect.

Decomposition by Occupation (Postel-Vinay and Robin, 2002)

LOG WAGE VARIANCE DECOMPOSITION

Occupation	Nobs.	Mean log wage: $E(\ln w)$	Total log-wage variance/coeff. var. $V(\ln w)$	CV	Case $U(w) =$	Firm effect: $VE(\ln w \mid p)$ Value	% of $V(\ln w)$	Search friction effect: $EV(\ln w \mid p) - V \ln \varepsilon$ Value	% of $V(\ln w)$	Person effect: $V \ln \varepsilon$ Value	% of $V(\ln w)$
Executives, manager, and engineers	555,230	4.81	0.180	0.088	$\ln w$	0.035	19.3	0.082	45.5	0.063	35.2
					w	0.035	19.4	0.070	38.7	0.076	41.9
Supervisors, administrative and sales	447,974	4.28	0.125	0.083	$\ln w$	0.034	27.5	0.065	52.1	0.025	20.3
					w	0.034	27.9	0.069	55.1	0.022	17.8
Technical supervisors and technicians	209,078	4.31	0.077	0.064	$\ln w$	0.025	32.4	0.044	57.6	0.008	10.0
					w	0.025	32.8	0.047	60.6	0.005	6.6
Administrative support	440,045	4.00	0.082	0.072	$\ln w$	0.029	35.7	0.043	52.2	0.010	12.1
					w	0.028	34.6	0.045	55.7	0.008	9.7
Skilled manual workers	372,430	4.05	0.069	0.065	$\ln w$	0.029	42.9	0.039	57.1	0	0
					w	0.028	41.5	0.040	58.5	0	0
Sales and service workers	174,704	3.74	0.050	0.060	$\ln w$	0.020	40.8	0.029	58.7	0.0002	0.4
					w	0.019	37.1	0.029	57.9	0.0025	5.0
Unskilled manual workers	167,580	3.77	0.057	0.063	$\ln w$	0.027	48.3	0.029	51.7	0	0
					w	0.023	40.8	0.033	59.2	0	0

Job-Stayers Wage Growth (yearly, Postel-Vinay and Robin, 2002)

DYNAMIC SIMULATION YEARLY VARIATION IN REAL WAGE WHEN HOLDING THE SAME JOB OVER THE YEAR

Occupation	Case	Median Δlog wage (%)	% obs. such that Δlog wage $\leq$ −0.10	−0.05	0	0.05	0.10
Executives, managers, and engineers	$U(w) = \ln w$	0	0	0	85.8	93.9	96.6
	$U(w) = w$	0	0	0	84.2	93.7	96.8
Supervisors, administrative, and sales	$U(w) = \ln w$	0	0	0	84.7	94.8	97.3
	$U(w) = w$	0	0	0	84.5	95.1	97.3
Technical supervisors and technicians	$U(w) = \ln w$	0	0	0	87.2	95.8	97.9
	$U(w) = w$	0	0	0	85.9	96.1	98.1
Administrative support	$U(w) = \ln w$	0	0	0	84.9	94.7	97.3
	$U(w) = w$	0	0	0	82.9	94.9	97.2
Skilled manual workers	$U(w) = \ln w$	0	0	0	85.6	94.5	97.2
	$U(w) = w$	0	0	0	83.7	94.2	96.8
Sales and service workers	$U(w) = \ln w$	0	0	0	84.0	94.9	97.5
	$U(w) = w$	0	0	0	82.8	94.8	97.4
Unskilled manual workers	$U(w) = \ln w$	0	0	0	84.5	94.2	96.8
	$U(w) = w$	0	0	0	82.6	94.4	97.3

Job-to-Job Wage Growth (yearly, Postel-Vinay and Robin, 2002)

DYNAMIC SIMULATION VARIATION IN REAL WAGE AFTER FIRST RECORDED JOB-TO-JOB MOBILITY

Occupation	Case	Median Δlog wage (%)	% obs. such that Δlog wage $\leq$				
			−0.10	−0.05	0	0.05	0.10
Executives, managers, and engineers	$U(w) = \ln w$	3.1	13.0	22.9	38.8	55.1	65.4
	$U(w) = w$	3.7	7.9	17.3	34.9	54.0	65.1
Supervisors, administrative, and sales	$U(w) = \ln w$	3.3	2.7	12.4	35.0	55.8	66.7
	$U(w) = w$	2.6	3.3	11.2	34.2	57.9	69.7
Technical supervisors and technicians	$U(w) = \ln w$	2.8	4.2	10.0	32.2	57.8	71.8
	$U(w) = w$	3.9	2.9	9.0	34.2	54.8	69.3
Administrative support	$U(w) = \ln w$	5.1	1.1	6.1	24.3	49.7	64.4
	$U(w) = w$	5.3	1.0	5.2	24.0	49.2	63.8
Skilled manual workers	$U(w) = \ln w$	4.5	1.7	7.5	28.2	51.7	66.0
	$U(w) = w$	4.4	4.3	12.4	30.6	51.7	64.7
Sales and service workers	$U(w) = \ln w$	3.0	0.2	5.5	31.0	59.1	75.3
	$U(w) = w$	3.4	2.0	8.2	30.7	57.2	75.1
Unskilled manual workers	$U(w) = \ln w$	3.6	0.2	4.4	29.4	55.5	70.0
	$U(w) = w$	2.7	1.0	7.3	32.4	58.6	70.0

Next Time

- Next Tuesday: Equilibrium search and matching: Mortensen-Pissarides.
- 9/24: turn in your research proposal/introduction (email and cluster).