

Quantitative Macro-Labor: General Equilibrium Search and Matching

Professor Griffy

UAlbany

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Announcements

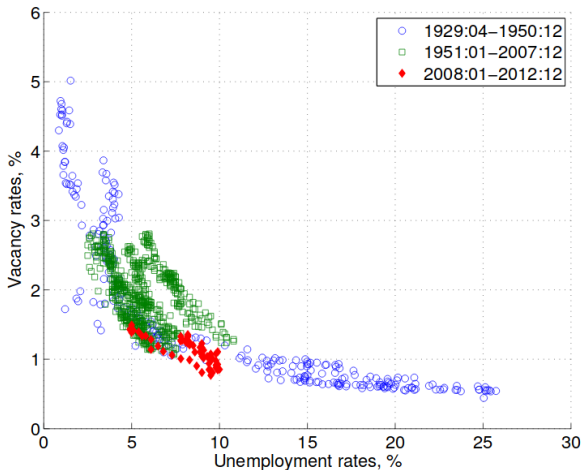
- ▶ Today: the Mortensen and Pissarides model (canonical equilibrium search)
- ▶ Everyone should have started the data project.
- ▶ Tuesday: turn in research introduction/proposal/outline.

Arrival Rates of Job Offers

- ▶ So far, we have assumed that the arrival rate of job offers is *exogenous*: regardless of equilibrium, the frequency with which you receive an offer is the same.
- ▶ Consider an example:
 1. There is a productivity downturn:
 2. How does a firm respond?
 3. BM, (BC), PVR: the quality of the offer distribution deteriorates, but searchers receive offers at the same rate.
- ▶ Essentially, slackness in the labor market is due to worker selectivity, not due to decisions made by the firm.
- ▶ Obviously, firms do respond.

The Beveridge Curve

- ▶ Another implication: there is no relationship between unemployment and vacancy creation.



(source: Petrosky-Nadeau and Zhang, 2017)

Hazard vs. Arrival Rate

- ▶ Unmatched firm problem in Burdett-Mortensen:

$$\pi = \max_w (p - w) l(w | w_R, F)$$

- ▶ The hazard rate is an equilibrium object; the arrival rate is not.
- ▶ But what if firms can adjust along the extensive margin in addition to the intensive margin?
- ▶ “Vacant” firm’s problem:

$$\pi^V(w) = -\kappa + q(.)J(w)$$

- ▶ q is the worker-finding rate. Now an equilibrium object.

The DMP Model (“Ch. 1 of Pissarides (2000)”)

► Agents:

1. Employed workers;
2. unemployed workers;
3. vacant firms;
4. matched firms.

► Linear utility ($u = b, u = w$) and production $y = p > b$.

► Matching function:

1. Determines *number* of meetings between firms & workers.
2. Args: levels searchers & vacancies ($U = u \times L, V = v \times L$)
3. Constant returns to scale (L is lab. force):

$$M(uL, vL) = uL \times M\left(1, \frac{v}{u}\right) = uL \times p(\theta)$$

4. where $\theta = \frac{v}{u}$ is “labor market tightness”
5. Match rates:

$$\underbrace{p(\theta)}_{\text{Worker}} = \theta \underbrace{q(\theta)}_{\text{Firm}}$$

Worker Value Functions

- ▶ Value functions:
 1. Employed at wage w : $W(w)$
 2. Unemployed: U .
- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

- ▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

Firm Value Functions

- ▶ Value functions:
 1. Filled, paying wage w : $J(w)$
 2. Vacant V .
- ▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V]$$

- ▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

- ▶ Free entry equilibrium condition:

$$\begin{aligned} rV &= 0 \\ \rightarrow \frac{\kappa}{E[J(w)]} &= q(\theta) \end{aligned}$$

- ▶ This is just a market clearing condition!

Equilibrium Objects

- ▶ Three key equilibrium objects:
 1. Wages;
 2. unemployment;
 3. $\theta = \frac{v}{u}$ (vacancies).
- ▶ How we determine each of these is largely a modeling decision.
- ▶ Steady-state: pin down unemployment via flow equation.
- ▶ Free-entry: Assume that firms always post vacancies so that free entry binds.
- ▶ Wages: Assume that wages are determined by a surplus-(profit) sharing rule.

Steady-State Unemployment

- Flow of unemployment:

$$\dot{u} = \delta(1 - u) - p(\theta)u$$

- Steady-state:

$$0 = \delta(1 - u) - p(\theta)u$$

$$p(\theta)u = \delta(1 - u)$$

$$u = \frac{\delta}{\delta + p(\theta)}$$

- Same as McCall with $\alpha = p(\theta)$.
- (Note: no heterogeneity & $p > b \rightarrow$ all wages accepted.)

Free Entry

- ▶ Free entry $V = 0$:

$$\begin{aligned}rJ(w) &= (p - w) + \delta[\cancel{V} - J(w)] \\(r + \delta)J(w) &= (p - w)\end{aligned}$$

- ▶ Vacancy creation condition (i.e., free entry imposed):

$$\begin{aligned}q(\theta) &= \frac{\kappa}{E[J(w)]} \\q(\theta) &= \frac{\kappa(r + \delta)}{(p - w)} \\\theta &= q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)\end{aligned}$$

- ▶ Thus, mapping between wages and θ . 1 equation, 2 unknowns.
- ▶ Need equation to determine wages in equilibrium.

Wage Determination

- ▶ Previous wage determination approaches:
 1. Burdett-Mortensen: wage-posting game, contracts non-negotiable.
 2. Postel-Vinay & Robin: wage-posting game, contracts set wages to equalize value of outside offer.
- ▶ Here: workers and firms bargain over the surplus of a match.
- ▶ Surplus of a match:

$$S(w) = W(w) + J(w) - U - \cancel{V}$$

$$S(w) = W(w) + J(w) - U$$

- ▶ Nash Bargaining splits this surplus according to a bargaining weight, β :

$$w = \operatorname{argmax}_w \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

Wage Determination

- Nash Bargaining splits this surplus according to a bargaining weight, β :

$$w = \operatorname{argmax}_w \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$\begin{aligned} 0 &= \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta} \frac{\partial W}{\partial w} \\ &\quad + (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U) \frac{\partial J}{\partial w} \end{aligned}$$

- $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\beta \left(\frac{J(w)}{W(w) - U} \right)^{1-\beta} = (1 - \beta) \left(\frac{W(w) - U}{J(w)} \right)^\beta$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

Wage Determination

- Nash Bargaining splits this surplus according to a bargaining weight, β :

$$w = \underset{w}{\operatorname{argmax}} \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$w \text{ solves } (W(w) - U) = \beta(W(w) + J(w) - U) = \beta S(w)$$

- Plug in for each of these:

$$(1 - \beta)[W(w) - U] = \beta J(w)$$

$$\begin{aligned} \beta J(w) &= (1 - \beta)[w - \delta(U - V(w)) \\ &\quad - b - p(\theta)(W(w) - U)] \end{aligned}$$

$$(1 - \beta)(w - b) = \beta J(w) + (1 - \beta)(p(\theta) + \delta)[W(w) - U]$$

$$\begin{aligned} (1 - \beta)(w - b) &= \beta(p - w - \delta J(w)) \\ &\quad + (1 - \beta)(p(\theta) + \delta)[W(w) - U] \end{aligned}$$

Wage Determination

- Note that $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

- And $(1 - \beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1 - \beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta \frac{J(w)}{1 - \beta} \\ w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

- Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Computation

- ▶ How would we solve this model?
- ▶ Need way to compute three equilibrium objects:
 1. Wages;
 2. unemployment;
 3. $\theta = \frac{v}{u}$ (vacancies).
- ▶ How we determine each of these is largely a modeling decision.
- ▶ Steady-state: pin down unemployment via flow equation.
- ▶ Free-entry: Assume that firms always post vacancies so that free entry binds.
- ▶ Wages: Assume that wages are determined by a surplus-(profit) sharing rule.
- ▶ Computation:
 - ▶ Wages, vacancies: depend on surplus.
 - ▶ Unemployment: law of motion.
- ▶ Here: add aggregate shocks.

Worker Value Functions

- ▶ Value functions:
 1. Employed at wage w : $W(w)$
 2. Unemployed: U .
- ▶ Unemployed flow value:

$$rU(z) = b + p(\theta)E[W(w, z) - U(z)] + \gamma E[U(z') - U(z)]$$

- ▶ Employed flow value:

$$\begin{aligned} rW(w, z) = & w(z) + \delta[U(z) - W(w, z)] \\ & + \gamma E[W(w', z') - W(w, z)] \end{aligned}$$

Firm Value Functions

► Value functions:

1. Filled, paying wage w : $J(w)$
2. Vacant V .

► Vacant flow value:

$$rV(z) = -\kappa + q(\theta(z))E[J(w, z) - V(z)] + \gamma[V(z') - V(w, z)]$$

► Matched flow value:

$$\begin{aligned} rJ(w, z) = & (z + p - w) + \delta[V(z) - J(w, z)] \\ & + \gamma[J(w', z') - J(w, z)] \end{aligned}$$

► Free entry equilibrium condition:

$$\begin{aligned} rV &= 0 \\ \rightarrow \frac{\kappa}{E[J(w, z)]} &= q(\theta) \end{aligned}$$

Computation

- Surplus of a match:

$$S(w, z) = W(w, z) + J(w, z) - U(z) - \cancel{V(z)}$$

$$S(w, z) = W(w, z) + J(w, z) - U(z)$$

- Plugging in and using $\beta S(w, z)$ is workers surplus and $(1 - \beta)S(w, z)$ is firm surplus:

$$S(z) = \frac{p + z}{r + \delta + \gamma} - \frac{b + \theta \kappa \frac{\beta}{1 - \beta}}{r + \delta + \gamma} + \frac{\gamma}{r + \delta + \gamma} \int_{z'} S(x) dF(x)$$

- This is just a contraction: $\frac{\gamma}{r + \delta + \gamma} < 1$.
- Pick $S_0(z_i) = 0, \forall i$ and iterate.
- Yields vacancies $q(\theta) = \frac{\kappa}{(1 - \beta)S(z)}$ and wages ($w = \beta S(z)$).
- Solve discrete time version for homework due Thursday after next (10/6).
- Then simulate unemployment dynamics for 1000 periods.

Conclusion

- ▶ Next: Research proposals due.
- ▶ Thursday: Endogenous separation? **Hosios Condition?**
Directed Search?
- ▶ Start your data projects.