

Quantitative Macro-Labor: Inequality in Heterogeneous Agent Models

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Announcements

- ▶ Today: extension of Block Recursive model with human capital and assets
- ▶ How does this affect inequality? (Griffy, 2021)
- ▶ Empirical regularities project is due in two weeks!
- ▶ Data projects due 11/3.
- ▶ Data projects presentations 10/27 and 10/29.

Wealth and Borrowing Constraints

- ▶ Low wealth limits ability to borrow early in the life-cycle.
- ▶ Feared or were denied credit (ages 20-30):
 - ▶ 1st quartile (Survey of Consumer Finances, 2013): 50%
 - ▶ Rest of population (SCF, 2013): 33%
- ▶ Less likely to be able to borrow in the future (ages 20-30):
 - ▶ 1st quartile (SCF, 2013): unsecured 80% of total debt
 - ▶ Population Average (SCF, 2013): unsecured 41% of total debt
- ▶ Wealth and earnings are correlated:
 - ▶ Low wealth, lower initial earnings;
 - ▶ Lower slope over life-cycle.

Question

- ▶ How do **differences** in **wealth**, **human capital**, and **learning ability** at **labor market entry** impact life-cycle
 - ▶ job search behavior?
 - ▶ human capital accumulation?
 - ▶ consumption?
- ▶ What channels are quantitatively important?

What I Do

- ▶ Construct quantitative general equilibrium life-cycle model:
 - ▶ search and matching in the labor market;
 - ▶ risk-aversion and borrowing constraints;
 - ▶ endogenous human capital accumulation.
- ▶ Estimate model using indirect inference.
- ▶ Consider counterfactual initial conditions.
- ▶ Decompose effect into interaction between wealth, search, and human capital.

Model Environment

- ▶ Life-cycle model: age discrete, indexed by t ; retire at $T + 1$.
- ▶ Agents:
 - ▶ Employed and unemployed workers.
 - ▶ Matched and unmatched firms.
- ▶ Technology:
 - ▶ Frictional matching in labor markets.
 - ▶ Endogenous human capital accumulation.
 - ▶ Borrowing constraints.
- ▶ Initial heterogeneity:
 - ▶ Initial wealth (a_0), human capital (h_0), and learning ability (ℓ).

Agents

- ▶ Risk-averse workers indexed by (a, h, ℓ, t) :
 - ▶ Employed (μ), unemployed w/ UI (b_{UI}) or w/o UI (b_L).
 - ▶ Search on and off job.
 - ▶ Consume & save s.t. borrowing constraint $a' \geq \underline{a}_t$.
 - ▶ Emp.: portfolio allocation (HC inv. & precautionary savings).
 - ▶ Unemployed & employed: stochastic HC depreciation.
- ▶ Continuum of profit maximizing firms:
 - ▶ Risk neutral. Produce using human capital.
 - ▶ Post vacancies that specify piece-rate μ .
- ▶ World risk-free rate r_F ; common discount rate β .
- ▶ Type-distribution $\phi' = \Phi(\phi)$ (suppressed throughout).

Search and Matching Technology

- ▶ Directed search (Moen, 1997):
 - ▶ Submarket: homogeneous workers (a, h, ℓ, t) and firms (μ)
 - ▶ Workers apply to job in submarket w/ known piece-rate μ .
- ▶ Matching technology:
 - ▶ # of matches in submkt (μ, a, h, ℓ, t) : $M_t = M(s_t, v_t)$ (CRS).
 - ▶ Submarket tightness: $\theta_t(\cdot) = \frac{v_t}{s_t}$
 - ▶ Worker finding rate: $q(\theta_t) = \frac{M(s_t, v_t)}{v_t}$
 - ▶ Job finding rates: $p(\theta_t) = \frac{M(s_t, v_t)}{s_t} = \theta_t q(\theta_t)$

Firms

- ▶ States: $s_J = (\mu, a, h, \ell)$, $s' = (\mu', a', h', \ell)$, $s'_J = (\mu, a', h', \ell)$
- ▶ Matched firms:
 - ▶ produce $(1 - \tau)h$, pay $\mu(1 - \tau)h$
 - ▶ separate exog. w/ prob. δ ; endog. w/ prob. $\lambda_E p(\theta_t(s'))$
 - ▶ continue w/ value $J_{t+1}(s'_J)$
- ▶ Value of filled vacancy with age- t type- s_J worker:

$$J_t(s_J) = (1 - \mu)(1 - \tau)h + \beta E[(1 - \delta)(1 - \lambda_E p(\theta_t(s')))]J_{t+1}(s'_J)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau))$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

- ▶ Worker decisions: μ', a', h', τ .

Free Entry and Equilibrium Job-Finding Rates

- ▶ Unmatched firms:
 - ▶ Pay κ to post (profitable) vacancies.
 - ▶ Match w/ prob. $q(\theta_t(s_J))$.
- ▶ Value of vacancy with age- t type- s_J worker:

$$V_t(s_J) = -\kappa + q(\theta_t(s_J))J_t(s_J)$$

- ▶ Free Entry ($V_t(s_J) = 0$):

$$q(\theta_t(s_J)) = \frac{\kappa}{J_t(s_J)}$$
$$\theta_t(s_J) = q^{-1}\left(\frac{\kappa}{J_t(s_J)}\right)$$

- ▶ Eqm. job finding rate: $p(\theta_t) = \theta_t q(\theta_t)$ determined by J_t, κ
- ▶ Eqm.: $\frac{\partial P}{\partial \mu} < 0$

Unemployed Searcher's Problem

- ▶ States (w/ UI): $s_U = (b_{UI}, a, h, \ell)$, $s'_E = (\mu', a, h, \ell)$
- ▶ States (w/o UI): $s_U = (b_L, a, h, \ell)$, $s'_E = (\mu', a, h, \ell)$
- ▶ Unemployed searcher's problem:
 - ▶ Apply for job w/ piece-rate μ' .
 - ▶ Transition to employment w/ prob. $p(\theta_t(s'_E))$.
 - ▶ Continue w/ value $W_t(s'_E)$ if offered job.
 - ▶ Continue w/ value $U_t(s_U)$ if no offer.
- ▶ Value of searching while unemployed:

$$R_t^U(s_U) = \max_{\mu'} p(\theta_t(s'_E)) W_t(s'_E) + (1 - p(\theta_t(s'_E))) U_t(s_U)$$

Unemployed Searcher's Problem

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- ▶ Competitive labor market:
 - ▶ Paid marginal product $\rightarrow$ inc. inequality because of diffs in HC
 - ▶ Idiosyncratic shocks $\rightarrow$ consumption risk. Insurance via $a - \underline{a}$.
- ▶ Frictional labor market:
 - ▶ Frictions $\rightarrow \mu < 1$.
 - ▶ Employment risk $\rightarrow$ consumption risk.
 - ▶ Precautionary savings (& UI) only explicit insurance.
 - ▶ Alternative: decrease μ . $\rightarrow$ (low) wealth can impact earnings.

Unemployed Worker's Problem

- ▶ States:
 - ▶ Unemp. w/ UI: $s_U = (b_{UI}, a, h, \ell)$, $s'_{UI} = (b_{UI}, a', h', \ell)$
 - ▶ Unemp w/o UI: $s_U = (b_L, a, h, \ell)$, $s'_L = (b_L, a', h', \ell)$
- ▶ Consumption and savings problem:
 - ▶ Consume & save s.t. $a' \geq \underline{a}_t$.
 - ▶ Lose benefits w/ prob. γ .
 - ▶ Human Capital depreciates: $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$.
- ▶ Value of unemployment (w/ UI):

$$U_t(s_U) = \max_{c, a' \geq \underline{a}_t} u(c) + \beta E[(1 - \gamma)R_{t+1}^U(s'_{UI}) + \gamma R_{t+1}^U(s'_L)]$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI}$$

$$h' = e^{\epsilon'} h$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

Unemployed Worker's Problem

- States:

- Unemp. w/ UI: $s_U = (b_{UI}, a, h, \ell)$, $s'_{UI} = (b_{UI}, a', h', \ell)$

- Unemp w/o UI: $s_U = (b_L, a, h, \ell)$, $s'_L = (b_L, a', h', \ell)$

- Value of unemployment (w/ UI):

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$$h' = e^{\epsilon'} h$$

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Employed Worker's Problem

- ▶ States:

- ▶ Emp.: $s_E = (\mu, a, h, \ell)$, $s'_E = (\mu, a', h', \ell)$

- ▶ Unemp. w/ UI: $s'_U = (b_{UI}, a', h', \ell)$

- ▶ Employed Worker's Problem:

- ▶ Portfolio alloc.: $(a' \geq a_t, \tau)$, τ to HC inv. & $(1 - \tau)$ to work.

- ▶ Stochastic HC depreciation $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$

- ▶ Lose job w/ prob. δ , receive $b(1 - \tau)\mu h$.

- ▶ Value of employment:

$$W_t(s_E) = \max_{c, a' \geq \underline{a}_t, \tau} u(c) + \beta E[(1 - \delta)R_{t+1}^E(s'_E) + \delta R_{t+1}^U(s'_U)]$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + (1 - \tau)\mu h$$

$$b_{UI} = b(1 - \tau)\mu h$$

$$h' = e^{\epsilon'}(h + \ell(h\tau)^\alpha), \quad \epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

Employed Worker's Problem

$$W_t(s_E) = \max_{c, a' \geq \underline{a}_t, \tau} u(c) + \beta E[(1 - \delta)R_{t+1}^E(s'_E) + \delta R_{t+1}^U(s'_U)]$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + (1 - \tau)\mu h$$

$$b_{UI} = b(1 - \tau)\mu h$$

$$h' = e^{\epsilon'}(h + \ell(h\tau)^\alpha), \quad \epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$$

- ▶ Human capital inv. is risky:
 1. Rate of return uncertain: stochastic dep., unknown ex-ante.
 2. Illiquid: no consumption smoothing value when unemployed.
- ▶ Rate of return risk determines allocation for “wealthy-enough.”
- ▶ Separation while low-wealth $\rightarrow$ take low- μ job.
- ▶ $\rightarrow$ Exposure to unemployment risk distorts allocation.

Equilibrium

A *Block Recursive Equilibrium* (BRE) in this model is a set of value functions, $U_t, W_t, R_t^E, R_t^U, J_t, V_t$, associated policy and market tightness functions, a', c, μ', τ , and θ_t , which satisfy

1. The policy functions $\{c, \mu', a', \tau\}$ solve the workers problems, W_t, U_t, R_t^E, R_t^U .
2. $\theta_t(\mu, a, h, \ell)$ satisfies the free entry condition for all submarkets (μ, a, h, ℓ, t) .
3. The aggregate law of motion is consistent with all policy functions.

Estimation

- ▶ Indirect Inference (conditional MoM) (Gourieux et al, 1993):
 - ▶ Select reduced-form analogs to structural model.
 - ▶ Objective: match coefs. for regs. w/ data & simulated data.
 - ▶ Minimize by changing structural parameters.
- ▶ Basic approach:
 - ▶ Estimate effect of wealth on job search behavior.
 - ▶ Match age-earnings regs (eqm. outcome) by initial heterogeneity.
 - ▶ Match observable marginal distributions.

Empirical Preliminaries

- ▶ Quarterly model, ages 23-64, retire at 65.
- ▶ Model parameters: $\sigma = 2, r_F = 0.012, \beta = \frac{1}{1+r_F}$
- ▶ Power utility + unemp leisure: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$
- ▶ HC Evolution: $h' = e^\epsilon(h + H(h, \ell, \tau)) = e^\epsilon(h + \ell \times (h\tau)^\alpha)$
- ▶ Natural borrowing constraint: $\underline{a}_t = \sum_{j=t}^T \frac{b_L}{(1+r_F)^j}$
- ▶ Initial conditions:
 - ▶ $(a_0, h_0, \ell) \sim LN(\psi, \Sigma)$
 - ▶ Correlations $\rho_{AH}, \rho_{AL}, \rho_{HL}$
- ▶ Full list of preset values:

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Key Estimated Parameters and Coefficients

▶ Parameter Estimates

- ▶ Age-23 constraint: $\underline{a}_0 = -\$6,378$ (2011\$)
- ▶ HC curvature: $\alpha = 0.5687$.
- ▶ HC dep.: $(\mu_\epsilon, \sigma_\epsilon) = (-0.0249, 0.0621)$.
- ▶ Corrs.: $\rho_{AH} = 0.3253$ $\rho_{AL} = 0.4642$ $\rho_{HL} = 0.6915$.

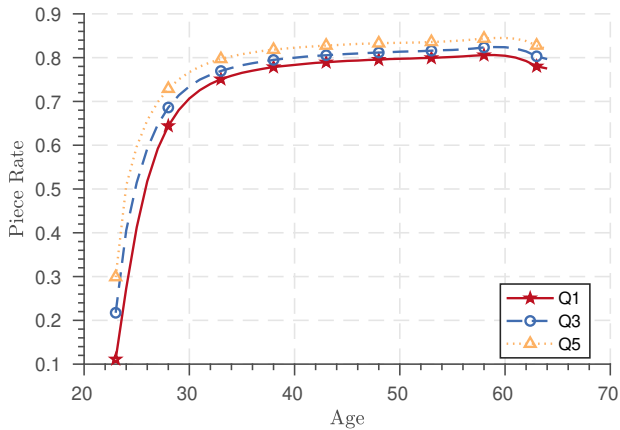
▶ Coefficient Estimates

- ▶ $\frac{\partial \ln(W_{i,j+1})}{\partial \ln(U_i)}$: Data: 0.4652; Model: 0.2918,
- ▶ $\frac{\partial \ln(W_{i,j+1})}{\partial \ln(U_i)} (q > 1)$: Data: -0.4425 ; Model: -0.2731
- ▶ $\frac{\partial \ln(H_{i,j+1})}{\partial \ln(U_i)} (q = 1)$: Data: -0.8664 ; Model: -0.932 ,
- ▶ $\frac{\partial \ln(H_{i,j+1})}{\partial \ln(U_i)} (q > 1)$: Data: -0.4542 ; Model: -0.3336
- ▶ ρ_{AH} : intercepts by wealth underpredicts higher quintiles.
- ▶ ρ_{AL} : overpredicts slopes by wealth in higher quintiles.
- ▶ ρ_{HL} : slopes by AFQT score quintile close.

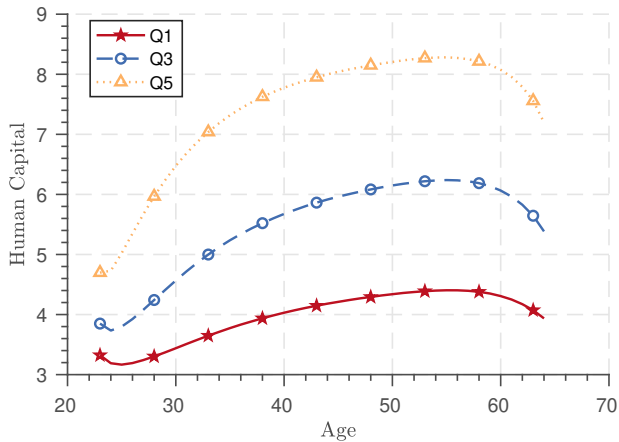
Findings

- ▶ Mechanisms & life-cycle earnings growth $w_t = \mu_t(1 - \tau_t)h_t$
- ▶ Two sources of earnings growth:
 - ▶ Movement up job (piece-rate) ladder. μ_t
 - ▶ Investment in human capital. h_t
- ▶ Consider two experiments, compare Inc., Cons., etc.:
 1. Decrease initial conditions of median worker by 1 SD for each (a_0, h_0, ℓ) .
 2. Eliminate initial dispersion for each (a_0, h_0, ℓ) .
- ▶ Decompose interaction between wealth, search, and human capital.

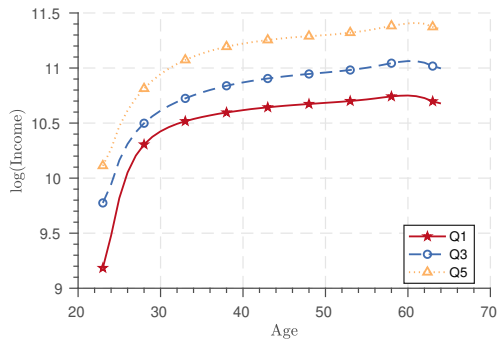
Job Ladder



Human Capital



Income

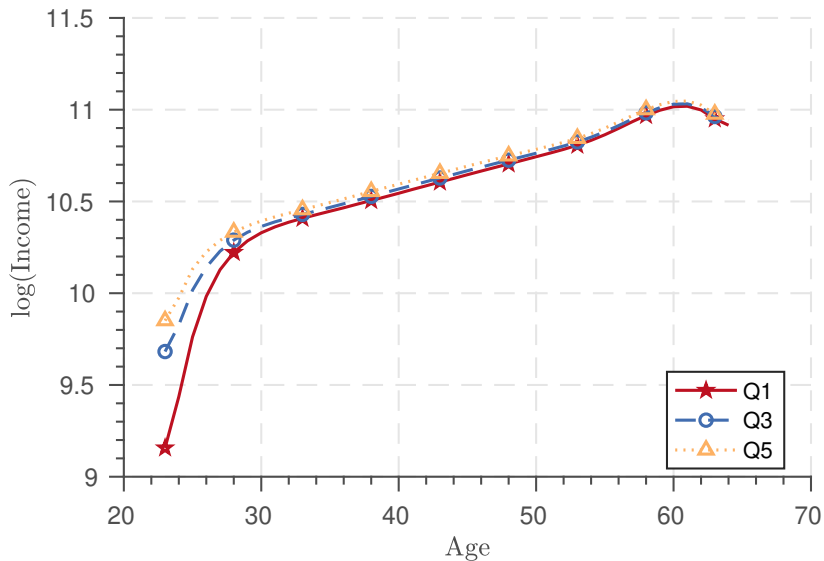


- ▶ Job ladder: important early.
- ▶ Human capital: important mid/late.

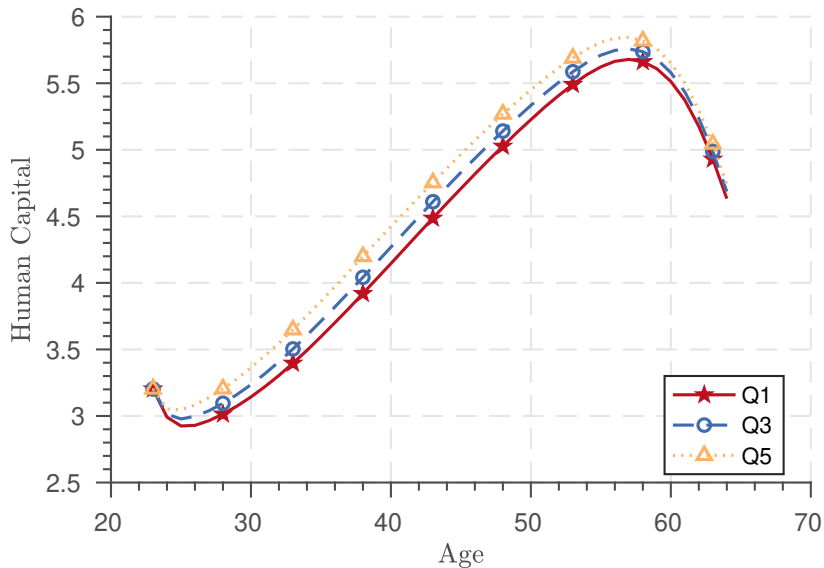
Sources of Inequality

- ▶ Explore 3 ways:
 1. Set h_0, ℓ to median initial value.
 - ▶ i.e., resulting variation due to wealth heterogeneity **only**.
 - ▶ Compare to previous figures.
 2. Subject median worker to -1 SD in each (a_0, h_0, ℓ) .
 - ▶ Same experiment as HVY (2011).
 3. Eliminate dispersion in initial conditions (separately).
- ▶ Focus on changes in average outcomes & by wealth.

Income



Human Capital



Findings: Median Worker

Change	Δ Consumption		Δ Earnings	Δh	$\Delta \tau$	$\Delta \mu'$
	(%)	HVY (%)	(%)	(%)	(%)	(%)
Wealth	-6.4	-1.6	-5.8	-2.5	-5.7	-4.8
Human Capital	-3.8	-28.3	-3.6	-4.8	-5.9	-0.4
Learning Ability	-15.5	-2.6	-16.8	-29.1	-96.3	0.3

Findings: No Dispersion

Counterfactual	Δ Income (%)				Δh (%)				$\Delta \mu$ (%)			
	1st	3rd	5th	Ave	1st	3rd	5th	Ave	1st	3rd	5th	Ave
$a_0 = E[a_0]$	5.79	1.09	-2.06	1.03	1.50	0.44	-1.33	0.12	5.44	0.89	-1.84	1.42
$h_0 = E[h_0]$	1.74	-0.65	-3.40	-1.10	3.16	0.69	-2.14	0.23	0.69	-0.16	-0.52	-0.01
$\ell = E[\ell]$	24.85	1.24	-17.97	-1.07	37.75	11.32	-8.37	9.65	1.26	-0.51	-1.35	-0.29

Decomposing the Interaction

- ▶ How does interaction between wealth, search, and human capital affect inequality?
- ▶ Compare outcomes in baseline model to 3 restrictions.
- ▶ Restrictions:
 - ▶ R1: exogenous portfolio $\tilde{\tau}_t(\mu, a, h, \ell) = \tau_t(\mu, \bar{a}_t, h, \ell) \forall t$ and $\tilde{a}'_t(\mu, a, h, \ell) = a_t(\mu, \bar{a}_t, h, \ell) \forall t$.
 - ▶ Bewley model: frictionless labor market, still human capital & savings decision.
 - ▶ R2: Bewley + exogenous portfolio $\tilde{\tau}_t(\mu, a, h, \ell) = \tau_t(\mu, \bar{a}_t, h, \ell) \forall t$ and $\tilde{a}'_t(\mu, a, h, \ell) = a_t(\mu, \bar{a}_t, h, \ell) \forall t$.

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- ▶ R1 - Base: precautionary effect on human capital by wealth in baseline model.
- ▶ R2 - Bewley: precautionary effect on human capital by wealth without frictional labor markets.
- ▶ Difference between these comparisons: interaction between wealth, search, human capital.

Findings: Exogenous Human Capital Comparison

Counterfactual	$\Delta \tau$ (%)					Δh (%)			
	1st	3rd	5th	Ave		1st	3rd	5th	Ave
% Δ (Base $\rightarrow$ R1)	33.18	17.84	6.42	16.51		6.01	4.90	1.36	4.09

Findings: Frictionless Labor Markets Comparison

Counterfactual	$\Delta\tau$				Δh			
	1st	3rd	5th	Ave	1st	3rd	5th	Ave
% Δ (Bewley $\rightarrow$ R2)	15.15%	12.49%	6.80%	11.16%	3.29%	3.75%	2.16%	3.19%
Effect of Wealth x Search	18.03pp	5.35pp	-0.37pp	5.35pp	2.72pp	1.16pp	-0.80pp	0.90pp

Findings: Interaction

Counterfactual	1st	3rd	5th
% Δ Income (Base $\rightarrow$ R1)	41.11%	3.24%	-26.87%
% Explained by Interaction	6.61%	35.69%	2.98%

Conclusion

- ▶ Constructed quantitative life-cycle model:
 - ▶ Risk-averse agents who face borrowing constraints.
 - ▶ General equilibrium labor market frictions.
 - ▶ Endogenous earnings growth through human capital choice.
- ▶ Estimated using indirect inference.
- ▶ Findings:
 - ▶ Borrowing constraints & search impact low-wealth individuals.
 - ▶ Wealth dynamically alters the earnings process through search behavior and human capital accumulation.
 - ▶ Initial wealth causes larger life-cycle changes than initial human capital (and sometimes learning ability).
- ▶ Don't forget to start your data projects