

Quantitative Macro-Labor: Efficiency in Search and Matching

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Announcements

- ▶ Today: the Hosios Condition
- ▶ Efficiency in search and matching models.
- ▶ Everyone should have started the data project.
- ▶ Data projects due 10/27.
- ▶ (Note: largely derived from Christine Braun's lecture on the DMP model).

Efficiency

- ▶ Is zero unemployment efficient? **No**
 - ▶ higher unemployment incentivizes firms to post vacancies
 - ▶ but high unemployment is costly, less production
- ▶ Is a high vacancy rate efficient?
 - ▶ vacancy creation is costly
 - ▶ but lots of vacancies reduces unemployment
- ▶ So what is the efficient level of θ ?

Efficiency

- ▶ Congestion externality
 - ▶ one more hiring firm makes unemployed workers better off and makes all other hiring firms worse off
 - ▶ one more searching worker makes hiring firms better off and makes all other searching workers worse off
- ▶ Appropriability
 - ▶ firm pays a cost κ to post vacancy but does not get to keep the entire output p

Efficiency

- ▶ What value of θ would the social planner choose to maximize total output/utility if he is constrained by the same matching frictions?
 - ▶ does not care about wage b/c it's a linear transfer from the firm to the worker
- ▶ Does there exist a wage such that job creation is the same in the decentralized equilibrium as in the social planners outcome?
- ▶ Can we achieve this wage with the Nash solution?

The DMP Model (“Ch. 1 of Pissarides (2000)”)

► Agents:

1. Employed workers;
2. unemployed workers;
3. vacant firms;
4. matched firms.

► Linear utility ($u = b, u = w$) and production $y = p > b$.

► Matching function:

1. Constant returns to scale (L is lab. force):

$$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta)$$

2. where $\theta = \frac{v}{u}$ is “labor market tightness”
3. Match rates:

$$\underbrace{p(\theta)}_{\text{Worker}} = \theta \underbrace{q(\theta)}_{\text{Firm}}$$

► Social planner: pick θ optimally, no need to respect free entry condition.

Social Planner's Problem

$$\int_0^{\infty} e^{-rt} [p(1-u) + bu - \kappa\theta u] dt$$

$$\text{s.t. } \dot{u} = \delta(1-u) - p(\theta)u$$

- ▶ Social planner's problem
 - ▶ $p(1-u)$: social output of employment
 - ▶ bu : leisure enjoyed by unemployed workers
 - ▶ $\kappa\theta u$: cost of jobs
- ▶ Social planner is subject to the same transition equation for unemployment

Social Planner's Problem

- The Hamiltonian

$$H = e^{-rt}[p(1-u) + bu - \kappa\theta u] + \mu(t)[\delta(1-u) - p(\theta)u]$$

- FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow -e^{-rt}\kappa u - \mu u(q(\theta) + \theta q'(\theta)) = 0$$

- μ : marginal value of an extra unemployed worker.

Social Planner's Problem

- Optimal θ

$$H_{\theta} = 0 \Rightarrow -e^{-rt}\kappa u - \mu u q(\theta) \left(1 + \frac{\theta q'(\theta)}{q(\theta)}\right) = 0$$

- What is $\frac{\theta q'(\theta)}{q(\theta)}$?

$$\begin{aligned} m(u, v) &= vq(\theta) \\ \rightarrow \frac{\partial m(u, v)}{\partial u} &= vq'(\theta) \frac{-v}{u^2} \\ \rightarrow \frac{\partial m(u, v)}{\partial u} &= -\theta^2 q'(\theta) \\ \rightarrow \frac{\frac{\partial m(u, v)}{\partial u}}{m(u, v)} &= -\frac{\theta^2 q'(\theta)}{vq(\theta)} \\ \rightarrow u \frac{\frac{\partial m(u, v)}{\partial u}}{m(u, v)} &= -\frac{\theta q'(\theta)}{q(\theta)} \end{aligned}$$

- $\frac{\theta q'(\theta)}{q(\theta)}$ is the elasticity of the matching function wrt u .

Social Planner's Problem

- ▶ The Hamiltonian

$$H = e^{-rt}[p(1 - u) + bu - \kappa\theta u] + \mu(t)[\delta(1 - u) - p(\theta)u]$$

- ▶ FOCs

$$H_u = -\dot{\mu} + r\mu \Rightarrow -e^{-rt}(p - b + \kappa\theta) - [\delta + r + p(\theta)]\mu + \dot{\mu} = 0$$

$$H_\theta = 0 \Rightarrow -e^{-rt}\kappa u - \mu u q(\theta)(1 - \eta(\theta)) = 0$$

- ▶ $\eta(\theta)$: elasticity of match fun. wrt u .

Optimal θ

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$):

$$\begin{aligned}\frac{p - b + \kappa\theta}{\delta + r + p(\theta)} &= \frac{\kappa}{q(\theta)(1 - \eta(\theta))} \\ (p - b)(1 - \eta(\theta)) + \kappa(1 - \eta(\theta))\frac{p(\theta)}{q(\theta)} &= \frac{(\delta + r + p(\theta))\kappa}{q(\theta)} \\ \rightarrow (1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa &= 0 \quad (1)\end{aligned}$$

- This is optimal θ

Decentralized solution

- ▶ Can the decentralized solution achieve the same level of θ ?
- ▶ i.e., can the decentralized level of unemployment be *efficient*?

Decentralized θ

- ▶ Free entry $V = 0$:

$$\begin{aligned}rJ(w) &= (p - w) + \delta[\cancel{V} - J(w)] \\(r + \delta)J(w) &= (p - w)\end{aligned}$$

- ▶ Vacancy creation condition (i.e., free entry imposed):

$$\begin{aligned}q(\theta) &= \frac{\kappa}{E[J(w)]} \\q(\theta) &= \frac{\kappa(r + \delta)}{(p - w)} \\\theta &= q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)\end{aligned}$$

- ▶ Thus, mapping between wages and θ . 1 equation, 2 unknowns.
- ▶ Need equation to determine wages in equilibrium.

Wage Determination

- Recall Nash Bargained wages:

$$w = \operatorname{argmax}_w \underbrace{(W(w) - U)^\beta}_{\text{Net Utility}} \underbrace{(J(w) - V)^{1-\beta}}_{\text{Net Profits}}$$

$$\begin{aligned} 0 &= \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta} \frac{\partial W}{\partial w} \\ &\quad + (1 - \beta)(J(w) - V)^{-\beta}(W(w) - U) \frac{\partial J}{\partial w} \end{aligned}$$

- $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\begin{aligned} \beta \left(\frac{J(w)}{W(w) - U} \right)^{1-\beta} &= (1 - \beta) \left(\frac{W(w) - U}{J(w)} \right)^\beta \\ \beta(J(w) + W(w) - U) &= W(w) - U \\ \beta S(w) &= W(w) - U \end{aligned}$$

Wage Determination

- Note that $\beta S(w) = [W(w) - U]$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta S(w)$$

- And $(1 - \beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1 - \beta}$

$$(1 - \beta)(w - b) = \beta(p - w - \delta J(w)) \\ + (1 - \beta)(p(\theta) + \delta)\beta \frac{J(w)}{1 - \beta} \\ w = (1 - \beta)b + \beta p + p(\theta)\beta J(w)$$

- Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

- Job creation curve:

$$(r + \delta)J(w) = (p - w)$$

$$q(\theta) = \frac{\kappa}{J(w)}$$

$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$

$$p - w - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- Now, plug in using wages we just found:

$$w = (1 - \beta)b + \beta p + \beta\theta\kappa$$

Decentralized free entry

- Job creation curve:

$$p - ((1 - \beta)b + \beta p + \beta \theta \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

- identities: $p(\theta) = \theta q(\theta) \rightarrow \theta = \frac{p(\theta)}{q(\theta)}$

$$\rightarrow p - ((1 - \beta)b + \beta p + \beta \frac{p(\theta)}{q(\theta)} \kappa) - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \beta \frac{p(\theta)}{q(\theta)} \kappa - \frac{\kappa(r + \delta)}{q(\theta)} = 0$$

$$(1 - \beta)(p - b) - \frac{r + \delta + \beta p(\theta)}{q(\theta)} \kappa = 0$$

- Looks familiar?

Social Planner's Problem

- Using $p(\theta) = \theta q(\theta)$ and solving in steady state ($\dot{\mu} = 0$)

$$(1 - \eta(\theta))(p - b) - \frac{\delta + r + \eta(\theta)p(\theta)}{q(\theta)}\kappa = 0 \quad (2)$$

- From the decentralized solution, plug the wage curve into the Job creation curve

$$(1 - \beta)(p - b) - \frac{\delta + r + \beta p(\theta)}{q(\theta)}\kappa = 0 \quad (3)$$

Efficiency

- ▶ Comparing (1) and (2) we see that we have efficiency in the decentralized market if $\beta = \eta(\theta)$. The workers bargaining power is equal to the elasticity of the matching function with respect to u .
- ▶ This is a general result: we have efficiency when

$$\eta(\theta) = \beta$$

- ▶ This is called the Hosios (1990) condition

Next Time

- ▶ Directed/competitive search.
- ▶ Data projects due 10/27.