

Quantitative Macro-Labor: Searching with Observable Posting Characteristics: Directed Search

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Announcements

- ▶ Today:
 1. Random vs. directed search
 2. Change primitive: workers can observe wage offers prior to match.
 3. i.e., Moen (1997)
- ▶ Be sure to start empirical regularities project.
- ▶ Data projects due 10/29.
- ▶ Data projects presentations 10/27 and 10/29.

Random vs. Directed Search

- ▶ How do workers find jobs?
- ▶ How much information do they have about a job before applying?
- ▶ Two extremes:
 1. Random Search: *no* information about a job prior to receiving offer.
 2. Directed Search: *all* information about a job prior to application.
- ▶ Why does this matter?
 1. Random search is generically inefficient: one worker may reject a job offer than another would accept.
 2. Directed search is generically efficient: by applying for a job, a worker signals that the job is already acceptable.
- ▶ As we will see next time, it also changes computational complexity.

Random vs. Directed Search II

- ▶ Empirically, how can we tell them apart?
- ▶ Hazard rate to wage w generically:

$$H_U(w) = \underbrace{\lambda(w)}_{\text{Arrival Rate}} \times \underbrace{f(w)}_{\text{Prob. Offer} = W} \quad (1)$$

- ▶ Random search:

$$H_U(w) = \underbrace{\lambda}_{\text{Arrival Rate}} \underbrace{[1 - F(w_R)]}_{\text{Selectivity}} \underbrace{f(w)}_{P(\text{Offer} = W)} \quad (2)$$

- ▶ Directed search:

$$H_U(w) = \underbrace{\lambda(w)}_{\text{Arrival}} \underbrace{[1 - F(w_R)]}_{F(w_R)=0} \underbrace{f(w)}_{P(\text{Offer} = w)} \quad (3)$$

$$= \underbrace{\lambda(W)}_{\text{Arrival}} \underbrace{f(w)}_{P(w=w_j)=1} \quad (4)$$

$$= \underbrace{\lambda(w)}_{\text{Arrival Rate of Wage } w} \quad (5)$$

Some Evidence

- ▶ (First couple Borrowed from Shouyong Shi)
- ▶ Hall and Krueger (08):
 1. 84% had information on wage prior to first interview.
- ▶ Holzer, Katz, and Krueger (91)
 1. Firms in high-wage industries receive more applications than low-wage industries, controlling for observables.
- ▶ Braun, Engelhardt, Griffy, and Rupert: unemployment insurance changes $\lambda(w) \rightarrow$ inconsistent with random search.

Mortensen and Pissarides Model

- ▶ Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U] \quad (6)$$

- ▶ Employed flow value:

$$rW(w) = w + \delta[U - W(w)] \quad (7)$$

- ▶ Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V] \quad (8)$$

- ▶ Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)] \quad (9)$$

- ▶ Free entry equilibrium condition:

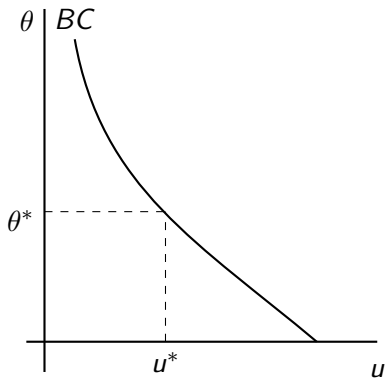
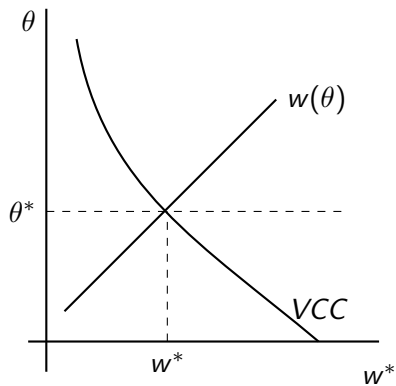
$$V = 0 \quad (10)$$

$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta) \quad (11)$$

Equilibrium

- ▶ The equilibrium we have described is a steady-state equilibrium characterized by value functions U, W, J, V , a wage function w , a market tightness function θ , and steady-state level unemployment u , such that
 1. A steady-state level of unemployment, derived from the flow unemployment equation.
 2. A wage rule that splits the surplus of a match according to a sharing rule with bargaining weight β
 3. A free entry condition that determines θ given wages and steady-state unemployment.
- ▶ What were these policy functions?
 1. $w = (1 - \beta)b + \beta p + \beta \theta \kappa$
 2. $\theta = q^{-1} \left(\frac{\kappa(r + \delta)}{p - w} \right)$
 3. $u = \frac{\delta}{\delta + p(\theta)}$

Equilibrium



Directed/Competitive Search

- ▶ In DMP, wages are negotiated/revealed after meeting.
- ▶ This can create inefficiency:
 1. Consider example with unemployed workers A and B.
 2. $w_R^A = 10$, $w_R^B = 15$
 3. Firm pays a cost κ to open a vacancy and posts a wage 12.
 4. Both worker A and B apply for the job. Firm randomly picks worker B.
 5. Worker B rejects job that would have been acceptable to worker A.
- ▶ Directed search: Worker B applies for different job with $w \geq w_R^B$.
- ▶ (Directed and competitive search generally used interchangeably).

The Competitive Search Model (Moen, 1997)

► Agents:

1. Employed workers employed in submarket i ;
2. unemployed workers considering searching in $i \in \{1, \dots, N\}$;
3. unmatched firms indexed by productivity $y_i \in y_1, \dots, y_N$;
4. matched firms indexed by productivity $y_i \in y_1, \dots, y_N$;
5. “Market Maker”: benevolent overlord who announces eqm. w_i .

► Linearity: ($u = z, u = w_i$) and $y = y_i > z$ in open submarkets.

► Matching function:

1. Determines *number* of meetings between firms & workers in submarket i :

$$M(u_i L_i, v_i L_i) = u_i L_i \times M(1, \frac{v_i}{u_i}) = u_i L_i \times p(\theta_i) \quad (12)$$

2. where $\theta_i = \frac{v_i}{u_i}$ is “submarket tightness”
3. Match rates:

$$\underbrace{p(\theta_i)}_{\text{Worker wage } i} = \theta_i \underbrace{q(\theta_i)}_{\text{Firm wage } i} \quad (13)$$

► i indexes both the productivity and wage.

Worker Value Functions

- ▶ Value functions:
 1. Employed in submarket i : W_i
 2. Unemployed and searching in submarket i : U_i .
 3. Unemployed: $U = \max\{U_1, \dots, U_N\}$.
- ▶ Unemployed flow value in submarket i :

$$rU_i = z + p(\theta_i)(W_i - U_i) \quad (14)$$

- ▶ Employed flow value in submarket i :

$$rW_i = w_i + \delta(U_i - W_i) \quad (15)$$

- ▶ Both problems are stationary: optimal choice of i true $\forall t$.

Worker Value Functions II

- We can solve for match rates:

$$rU_i = z + p(\theta_i)(W_i - U_i) \quad (16)$$

$$(r + p(\theta_i))U_i = z + p(\theta_i)\frac{w_i + \delta U_i}{r + \delta} \quad (17)$$

$$(r + \delta)(r + p(\theta_i))U_i - p(\theta_i)\delta U_i = (r + \delta)z + p(\theta_i)w_i \quad (18)$$

$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))} \quad (19)$$

$$p(\theta_i) = \frac{rU_i - z}{w_i - rU_i}(r + \delta) \quad (20)$$

$$(21)$$

- $U = \max\{U_1, \dots, U_N\}$ and ex-ante homogeneity among workers implies

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + \delta) \quad (22)$$

$$(23)$$

Firm Value Functions

- ▶ Firm observes own productivity, chooses to open vacancy given submarkets (w, θ) .
- ▶ Value functions:
 1. Vacant with productivity y_i : $V(y_i, w, \theta)$
 2. Filled with productivity y_i , paying wage w : $J(y_i, w)$
- ▶ Vacant flow value:

$$rV(y_i, w, \theta) = -\kappa + q(\theta)(J(y_i, w) - V(y_i, w, \theta)) \quad (24)$$

- ▶ Matched flow value:

$$rJ(y_i, w) = y_i - w + \delta(V(y_i, w, \theta) - J(y_i, w)) \quad (25)$$

Firm Value Functions II

- ▶ Value functions:
 1. Vacant with productivity y_i : $V(y_i, w, \theta)$
 2. Filled with productivity y_i , paying wage w : $J(y_i, w)$
- ▶ Moen assumes that $V(y_i, w, \theta) = 0$ in matched value only:

$$rJ(y_i, w) = y_i - w - \delta J(y_i, w) \quad (26)$$

- ▶ Asset value of vacancy in submarket (y_i, w, θ) :

$$(r + q(\theta))V(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + \delta} - \kappa \quad (27)$$

Equilibrium

- ▶ We will be interested in the same equilibrium objects, but now for each submarket i :
 1. Wages w_i ;
 2. unemployment u_i ;
 3. $\theta_i = \frac{v_i}{u_i}$ vacancies in each submarket.
- ▶ Before, 1 & 3 were separate equilibrium conditions.
- ▶ New equilibrium objects
 1. set of open submarkets, $\mathcal{I}$;
 2. value of unemployment $\bar{V}(U)$
- ▶ Market maker sets wages according to

$$\max_w V(y_i, w, \theta(w; U)) \quad (28)$$

- ▶ Given $p(\theta)$ from worker's problem, find w that maximizes value of vacancy.

Free Entry

- ▶ Free entry implies that the expected value of opening a vacancy will be equal to the cost of opening it κ
- ▶ The expected value of opening a vacancy

$$\bar{V}(U) = \sum_{i=\iota(U)}^n f_i V(y_i, w_i^*(U), \theta_i^*(U))$$

- ▶ equilibrium:

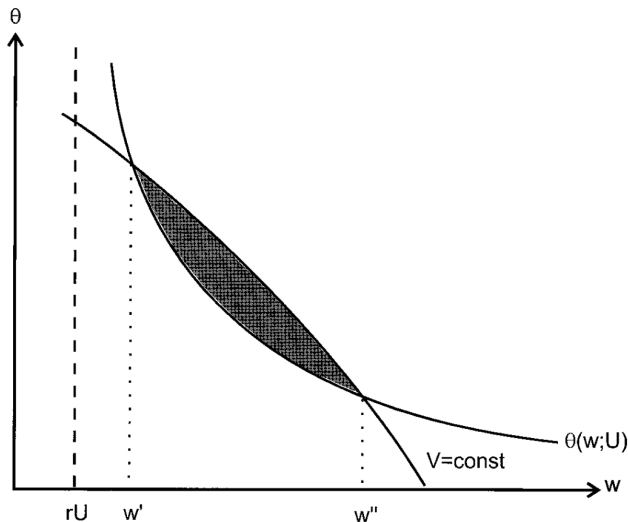
$$\bar{V}(U) = \kappa$$

Equilibrium Number of Markets

- ▶ We know that each productivity will form a separate market
- ▶ There are n productivities in the distribution
- ▶ All submarkets such that $w_i \geq rU$ will remain open
- ▶ Let ι denote the lowest submarket open

“Competitive” Search

- ▶ What is shaded region?
- ▶ (θ is worker indifference curve; V is firm)

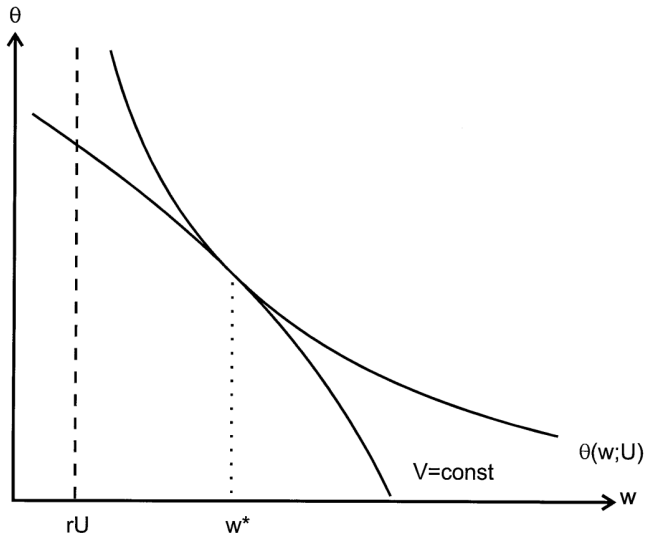


Directed vs. Random: Inefficiency

- ▶ Random search: search externalities/bargaining can trap markets at suboptimal allocation, because firms cannot commit to wages to attract workers.
- ▶ Directed search: firms post wages and compete for workers. In shaded region, a firm can post a wage $w \in (w', w'')$ that attracts workers while earning strictly higher profits.
- ▶ $\rightarrow$ equilibrium can only occur where V is tangent $\theta \rightarrow$ constrained efficiency (the Hosios condition).

“Competitive” Search

- Inefficiency (rejected matches in DMP)



Equilibrium

- The resulting competitive equilibrium with frictional labor markets is characterized by the following equations

$$\bar{V}(U) = \kappa \quad (29)$$

$$w_i = \arg \max V(y_i, w, \theta(w; U)), i \geq i_R \quad (30)$$

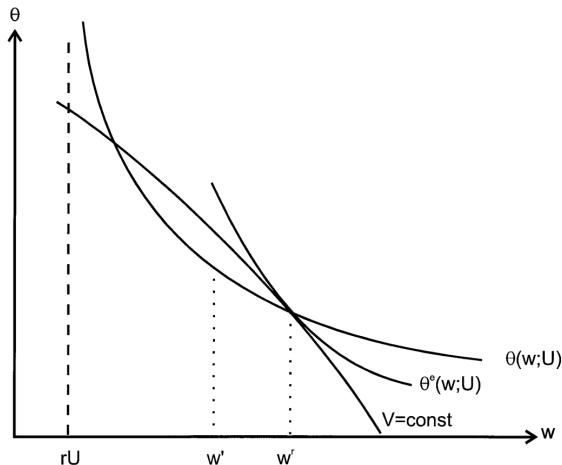
$$rU_i = \frac{(r + \delta)z + p(\theta_i)w_i}{(r + \delta + p(\theta_i))}, i \geq i_R \quad (31)$$

$$\dot{u}_i = 0; u_i p(\theta_i) = e_i \delta \quad (32)$$

$$\sum_i u_i = u \quad (33)$$

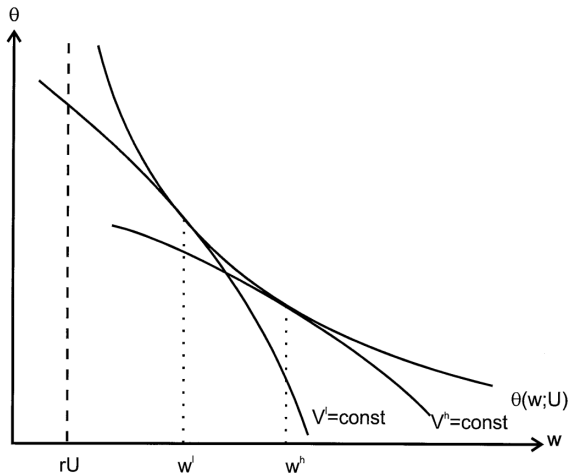
Wage Posting

- ▶ In previous description, wages were “announced in equilibrium” by a market maker.
- ▶ Would firms choose to deviate if they set their own wages?
- ▶ Suppose firms deviate and offer w' :



Wage Dispersion

- ▶ Absent any ex ante heterogeneity on the worker side, is there still wage dispersion?
- ▶ Workers indifferent between open submarkets in equilibrium:



Next Time

- ▶ Directed search with heterogeneity: block recursive equilibrium.
- ▶ Data projects due 10/29.